What I should have talked in Recitation 2: Some Linear Homogeneous ODE

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February 5, 2014

Disclaimer

- The slides are intended to serve as records for an informal talk (that never happened) in a recitation for math 244 course. It should never serve as any replacement for formal lectures or as any reviewing material. The author is not responsible for consequences brought by inappropriate use.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

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- Remark: Generally in 244 course, you are required to play with the second order case skillfully. So this requires that you can find the roots of a quadratic equation with more efficient ways.

A typical example

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Gosh, how am I supposed to know $\sqrt{441} = 21$? How am I supposed to know $19^2 = 361$?

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$r^2 + 19r - 5$

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The steps afterwards are much more time-consuming. Why not save some time in the first step?
So how to learn the skill?

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The End